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EDUCATION

INFLUENCE OF COMPUTATIONAL AND MECHANICAL ERRORS IN LOGARITHMS OF NUMBERS ON STUDENTS' PERFORMANCE IN MATHEMATICS IN PUBLIC, KENYA SECONDARY SCHOOLS IN MWALA SUB-COUNTY, MACHAKOS COUNTY, KENYA.

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ABSTRACT

Purpose of the Study: The objective of the study was to examine the effect of logarithmic calculation errors on students' performance in Mathematics in public secondary schools of Mwala sub-county, Machakos County, Kenya.

Methodology: This study employed a descriptive survey approach, targeting 6,720 students and 192 Mathematics teachers across 68 public secondary schools in Mwala Sub-County. A total of 672 students and 40 teachers from 20 randomly selected schools participated.

Findings: The findings of the study indicate that mechanical and computational mistakes significantly impact students' performance in mathematics tests on logarithms**.**

Conclusion: The study concludes that computational and mechanical errors significantly impact students' performance in mathematics tests on logarithms.

Recommendation: The study recommends that teachers should leverage these errors to develop interventions aimed at reducing them. Additionally, teachers have to be careful in the way they present logarithm concepts during the teaching process.

Keywords: *Mechanical errors, Computational errors, Mathematics, Logarithm, Performance*

INTRODUCTION

Concepts in Mathematics are understood through their structures and operations. These views are complementary as students solve problems from operational and structural approaches. This thinking perspective is embodied in fractions of logarithms, among other mathematical concepts. Sfard (1991) outlines that a student a Mathematical concept by moving from an operational view to the structure of a problem. When students make errors, it often indicates difficulties in grasping and applying certain concepts and techniques scientifically. Analyzing these mistakes can provide teachers with insights into students' understanding and approach to mathematical problems, as well as their incorrect problem-solving strategies.

Errors in mathematics arise for various reasons, broadly sourced from interpretations of operations and the structures of problems. Radatz (1980) highlights that logarithmic errors are a by-product of their prior knowledge and practice. The study showed that the errors can be systematic, persistent, and lasting for a while; they can be derived from specific difficulties students face when receiving and processing information in mathematics learning.

Khair et al. (2018) conducted a study in South Sulawesi, Indonesia, to characterize the errors that are committed by students when they solve algebraic problems. The study described conceptual and procedural errors in problem-solving. The findings showed that conceptual errors in algebra are due to students' misconceptions about some concepts. The students often made equivalence on many concepts at a go without checking the conditions, and students had challenges in interpreting the mathematical symbols.

Arigiyati et al. (2021) conducted a study in SMK Muhammadiyah, India, to examine the rate at which the students committed arithmetic, writing, and most common mistakes in solving the logarithms equations. The study results indicated that students committed arithmetic errors at 53% and writing errors at 34.78%. Similarly, Yodiatmana & Kartini (2022) aimed to describe errors that arise in solving logarithm problems. The study showed that the students significantly committed conceptual, mechanical, and technical errors.

The report by the WASSCE (2017) noted that some of the weaknesses students showed in their poor performance in Mathematics were in finding values in expressions of logarithms and translating the word problem to a mathematical expression that can be solved. Further, the Chief Examiner's (Elective 2) report showed that students had problems applying logarithm laws. The report indicated that students in Ghana's 2013 National examinations had computation challenges in solving logarithm problems such as $\log(x-2)+\log 2 = 2\log y$. The students failed to derive linear equations as they could not correctly apply the laws of logarithms.

Chege (2015)'s conducted a study in Kenya to assess errors committed by high school students while solving word problems. The specific objectives was to examine the source for errors, types of errors committed and the interventions to correct the challenges of errors. Using a cross-section research design, the study utilized a questionnaire to collect data from 32 secondary schools. The findings from the study indicated that students committed computational errors, wrongly developed equations and wrongly formulated problems.

Research on the common ways students make errors in Mathematics is growing. However, there has been less emphasis on the typical errors made by students in high school while handling issues involving logarithmic properties, real-world applications of the logarithm function, and logarithmic equations. This study intends to analyze the effects of mechanical and computational errors on students' performance in mathematics tests on the topic of the logarithm of numbers in public secondary schools in Mwala Sub-county, Kenya.

OBJECTIVE OF THE STUDY

The objective of the study was to examine the effect of logarithmic calculation errors on students' performance in Mathematics in public secondary schools of Mwala sub-county, Machakos County, Kenya.

RESEARCH HYPOTHESIS

The following hypotheses were developed for the study;

H_1.Computational errors do not significantly influence the student's performance in mathematics tests on the topic of the Logarithm of numbers in public secondary schools in the Mwala sub-county.

H_2.Mechanical errors do not significantly influence the student's performance in mathematics tests on the topic of logarithm of numbers in public secondary schools in the Mwala sub-county.

THEORETICAL FRAMEWORK

The study drew inspiration from Piaget's Theory of Intellectual Development (1967), which posits that the errors students frequently make can significantly impact their academic performance. According to this theory, a learner's errors are often reflective of their current stage of mental development, a concept further supported by Vygotsky in 2011. This perspective suggests that errors should be carefully analyzed to understand their underlying causes and the student's thought process. Piaget's framework also emphasizes that actions or answers that may appear illogical to an observer might actually make sense from the student's perspective. Therefore, it's crucial for educators to grasp the students' viewpoints that lead to these errors in order to devise effective interventions.

EMPIRICAL REVIEW

Mechanical errors in solving logarithmic problems can arise from a hurried approach, omitted steps, or failure to review previous work. Learners might make errors related to the order of operations due to reliance on memorization. In their rush to solve logarithm problems, students often confuse the quotient rules for logarithms. It is important to note, as demonstrated below, that the quotient property applies to two logarithms only when there is a change in the base formula.. For instance, students tend to commit mechanical errors due to forgotten steps in handling the quotients as below;

$$\mathrm{Loga}\,(\mathrm{M} - \mathrm{N}) \neq \frac{\mathrm{logaM}}{\mathrm{logaN}}$$

Mulungye et al., 2016, conducted a study in secondary schools in Machakos County - Kenya to examine various errors and misconceptions of students in algebra. The study examined the teachers' knowledge of errors and the strategies in place to correct the errors. The study adopted a descriptive research design, with a sample size of 432 students from form 2 and 15 Mathematics teachers. Some of the errors committed by the students were from the word problem (63%), and (22.3%) were sourced from equation problems. Students had computation challenges in solving variables and expressions at 39.6% and 40.9%, respectively. Similarly, Nelson & Powell (2017) indicated that students commit computational errors when trying to compute a problem using an erroneous number factor or an inappropriate/wrong operation, which leads to poor test scores in Mathematics subjects. For instance, in the following illustration, the function does not hold respect to sums and differences.

$$Loga\,(M \pm N) \neq loga\,M \pm loga\,N$$

According to Donuata & Pratama (2021), students also mix up what the product rule of logarithms requires them to do. For instance, in the illustration below, there is no product of a sum and no property for the product of two logarithms.

$$LOGa\ (M\ +\ N)\ \neq (LOGa\ M)(LOGa\ N)$$

Gusmania & Amelia's (2019) study showed that students' Mathematics test scores in Indonesia are still below the KKM (Minimum completeness Criteria), indicating that their achievement is modest. Logarithmic content is believed to be the most difficult topic for students in high school. Furthermore, it was mentioned that 81% of the pupils, particularly in the logarithmic topics, had scores lower than the KKM. Kastberg (2002) explored the challenges that high school students face while attempting to use logarithmic properties and solve problems involving the logarithm of numbers in mathematics subject. The study's findings showed that mechanical errors significantly impacted students' test scores in logarithms of numbers.

Students struggle when it comes to learning how to perform logarithmic calculation processes. Students struggle with logarithmic problems because they don't fully grasp the many kinds of logarithmic properties; instead, they rely on memorization (Yodiatmana & Kartini, 2022). The study above showed that students' errors in solving logarithmic issues based on procedural mistakes affected their scores in terms of overall mathematics performance.

According to Weber (2002), students understand exponents and logarithms in various ways. Students should mostly understand that exponential functions and domains are restricted to natural numbers. For instance, the exponents with positive integer coefficients (B^y) require that B be repeatedly multiplied by y times. Therefore a, students with limited computational action will be limited to understanding the exponent where only the integers are positive. In such a case, the students can only compute values and manipulate formulas and stop at that. When there are fractions, the student will have computational challenges, hence wrong solutions.

According to Breidenbach et al. (1992), students commit mechanical errors due to failed interiorizing of the action of the integers as a process. Students who understand processes can imagine the results of a logarithm problem without actually acting. Equally, they can reverse the process to obtain a new process as the start of the problem given to them by the teachers. By failing to check exponentiation as a function and attaching properties of the function in the process, they are likely to commit errors. For instance, 5^x is a positive function, where a student starts with the integer (1) and multiplies it by the positive; this will be an increasing function; if X increases by a specific number like one, 5^x will be doubling. Students can imagine the process obtained by taking a reverse of the exponentiation steps to have another process of taking logarithms.

Additionally, Perencevich et al. (2007) indicate that expressions such as 3^3 can be viewed differently in logarithms. First, it can be an external prompt for the students to multiply 3*3*3. The second view is that this can be an exponentiation of applying 3^3, the product 3 factors of three. Most high school students cannot view this problem this way. Therefore, for students to reduce errors, they must understand that b^x can be represented by x factors of b in properties of logarithms. Toth & Toth (2021) note that logarithm errors arise when students understand the exponential function when its domains have natural numbers. Errors arise when students are presented with fractions, negative integers, and irrational numbers.

Engelhart's (1982) research investigated the instructional implications of computational errors. The findings indicated that engaging in dialogue with students is a primary method for uncovering their thought processes. Thus, to mitigate computational errors, it is crucial for educators to introspectively analyze these mistakes. Engelhart (1982) further explained that computational errors might originate from a student's approach to tackling mathematical problems. Errors can occur due to various factors, such as misinterpretation of logarithmic symbols, misconceptions, inadequate engagement with the problem, or the use of incorrect procedures during logarithm calculations.

The study by Rafi & Retnawati (2018) acknowledges that computational errors in logarithms of numbers arise from distorted theorem and misuse of the available data. The findings indicated that students use the “log” as a variable, leading to computation errors. Additionally, the findings highlighted that misusing exponents leads to students making errors in solving problems related to the logarithm of numbers.

Wood (2005) examined the errors made by students when answering issues involving logarithm properties and equations. The study classified the errors the students had committed as follows: mechanical errors (38.71%), distorted theorem (24.19%), process errors (19.36%), and misuse of signs (17.74%). This means that mechanical errors in the topic of logarithm of numbers have a high prevalence of occurrence; hence, teachers need to be careful when handling the topic.

According to Wahyuni et al. (2019), in terms of mathematics learning difficulties in algebra, students are unsure of how to find the upper and lower limits of integration, and they still have not figured out how to substitute integrals and partial integrals; they don't understand the difference between definite and indefinite integrals, they multiply positive and negative numbers carelessly, they square functions incorrectly, they have trouble representing images, they can't figure out the basic formula for the volume of a solid of revolution, and they are inaccurate when it comes to operating algebraic expression. Wahyuni et al. (2019) concluded that this is the major source of poor performance in algebraic expression.

METHODOLOGY

The study used a quantitative methodology as it allowed the utilization of survey data from the respondents being examined**.** According to Mugenda and Mugenda (2003), survey research seeks to extract information that describes the phenomenon under study.

The study design allowed the researcher to generate numerical and narrative data to describe, record, and analyze the collected data.

FINDINGS

This section presents the findings about how these errors affect students' performance in Mwala Sub-county Mathematics.

Computational Errors' Influence on Students' Mathematics Performance

Table 1 shows an analysis of computational errors

Table 1. Computational Errors Influence Students' Performance in Mathematics

Response	Frequency	Percent
Agree	18	45.0
Disagree	1	2.5
Strongly Agree	18	45.0
Undecided	3	7.5

Total	**40**	**100.0**

Source: Researcher (2022)

The analysis in Table 1 shows that a significant number of teachers agreed that computational error influences the performance of students in mathematics tests. This was supported by 45% who strongly agreed and 45% who agreed to the response in the survey. The results show that computational errors have hugely affected the student's performance in mathematics tests on the topic of the logarithm of numbers. This means that the final score of their mathematics grades will be affected.

Hypothesis testing

The following ANOVA test was conducted to test the following developed hypothesis;

H_1.Computational errors do not significantly influence the Logarithm of numbers on students' performance in mathematics in the Mwala sub-county.

Table 2. ANOVA[a]

Model		**Sum of Squares**	**Df**	**Mean Square**	**F**	**Sig.**
1	Regression	.587	1	.587	.991	.026[b]
	Residual	22.508	38	.592		
	Total	23.096	39			

a. Dependent Variable: Mathematical Performance

b. Predictors: (Constant), Frequency of committing [Computational Errors]in Mathematical calculations

The findings indicate a p-value of .026, which is considered statistically significant. As a result, the null hypothesis was rejected. This demonstrates that computational errors have a significant impact on students' performance in mathematics tests covering logarithms in public secondary schools in Mwala Sub-county.

Students' Test on Computational Error

Question **(**$1.6^3 \times 1.6^{-7}$) was examined to check the element of computational error by the students in the sampled schools in Mwala Sub-county. From the sample of 672 students, 435 committed the errors, representing 64.73%, while the other group completed the tests successfully, as shown in Table 3.

Table 3. Student Findings on Computation Test

Category	Frequency	Percentage
Committed Errors	435	64.73%
Completed Successfully	237	35.27
Totals	**672**	**100**

Source: Researcher (2022)

Table 3 shows that 435 students, representing 64.73%, did not score marks in the question because they either computed the powers wrongly or multiplied the base, giving incorrect answers, as shown below.

$1.6^3 \times 1.6^{-7}$

$=1.6^{-7+-3}$

$=1.6^{4}$ (Wrong computation of negative power)

Or,

$1.6^3 x 1.6^{-7} = 2.56^{+3-7}$

$=2.56^{-4}$ (which shows the wrong computation of the base)

The correct working should have been as shown below;

$1.6^3 \times 1.6^{-7}$

$= 1.6^{3-7}$

$\underline{\underline{= 1.6^{-4}}}$

The error in the initial computation of powers occurred because the students incorrectly calculated the negative powers. They made a mistake by overlooking that the number 3 was raised to a positive power, leading to the errors. It's important to recognize that developing a strong understanding of how to compute powers can significantly reduce students' computational errors in logarithm problems. Ultimately, this enhanced understanding can contribute to better mathematics scores in this topic.

Mechanical Errors' Influence on Students' Performance in Mathematics

Table 4 shows the extent to which the teachers agree that mechanical errors affect students' performance in Mathematics tests on the topic of logarithm of numbers in public secondary schools in Mwala Sub-county.

Table 4. Influence of Mechanical Errors in Students' Mathematics Performance

Response	Frequency	Percent
Agree	14	35.0
Strongly Agree	18	45.0
Disagree	1	2.5
Strongly Disagree	1	2.5
Undecided	6	15.0
Total	**40**	**100.0**

Source: Field Data (2022)

The results from the analysis in Table 4 indicate that mechanical errors influence students' performance in mathematics. This is supported by 45% of the teachers who strongly agreed with the statement. Additionally, 35% agreed on the impact of mechanical errors on students' performance in mathematics on the topic of the logarithm of numbers. Thus, the results significantly imply that students who continuously commit mechanical errors have a high probability of poor performance in tests on the topic.

Hypothesis Testing

The researcher conducted ANOVA tests to test the following hypothesis; $H_{2.}$Mechanical errors do not significantly influence the student's performance in mathematics tests on the topic of logarithm of numbers in public secondary schools in the Mwala sub-county.

Table 5. ANOVA[a]

Model		Sum of Squares	Df	Mean Square	F	Sig.
1	Regression	.203	1	.203	.337	.015[b]
	Residual	22.893	38	.602		
	Total	23.096	39			

a. Dependent Variable: Mathematical Performance

b. Predictors: (Constant), Frequency of committing [Mechanical errors] in Mathematical calculations

From the ANOVA results above, the p-value obtained was .015[b], which is statistically significant. This led to the rejection of the null hypothesis. Hence, the conclusion is that mechanical errors significantly influence the performance of students in the logarithm of numbers in Mathematics.

Student's Findings on Mechanical Errors

Table 6 below shows the frequency of errors due to inappropriate procedures from the students

Table 6. Errors Committed by Students Due to Inappropriate Procedure

Category	Frequency	Percentage
Committed Errors	389	57.89%
Completed Successfully	283	42.11%
Totals	**672**	**100**

Source: Researcher (2022)

In the mechanical outcomes of the students, the use of the wrong procedure was found to be the significant cause at 57.89%. A significant number of the students computed the errors as follows;

Solve the following;

$$\frac{1.7^3 \times 1.7^2}{2^{-3} \times 2^{-4}}$$

$$= \frac{1.7^{3+2}}{2^{-3\pm-4}}$$

The example shows that the students had an incomplete procedure in which they had assumed that the – sign in both the powers of two (2^{-3}x2^{-4}) would lead to a positive power. This can happen when the students hurry to operate the powers to complete their test. The correct procedure is shown overleaf.

$$= \frac{1.7^{3+2}}{2^{-3+-4}}$$

$$= \frac{1.7^{5}}{2^{-7}}$$

Table 7. Errors committed by students due to wrong opening of brackets

Category	Frequency	Percentage
Committed Errors	378	56.25%
Completed Successfully	294	43.75
Totals	**672**	**100**

Source: Researcher (2022)

Errors committed by the students by the wrong opening of brackets represented 56.25% of the students being involved. The students were required to solve the following problem;

Simplify the expression.

$$\frac{2^3 \times 2^3}{(2x^2)^2}$$

$$= \frac{2^6}{2x^4}$$

The above answer was wrong due to the wrong opening of the parenthesis. The students failed to recognize that 2 in the brackets should have been squared. The correct workings should have been as shown below.

$$\frac{2^3 \times 2^3}{(2x^2)^2}$$

$$\frac{2^6}{4x^4}$$

In the subsequent example used, they wrongly opened the brackets. The power of 2 outside the brackets affects both the x values and the value of 2 within the brackets. This can be attributed to the tendency of students to follow or seek to combine the powers of the logarithms, forgetting their bases.

CONCLUSION

The study concludes that computational and mechanical errors significantly impact students' performance in mathematics tests on logarithms. The study concludes that with a p-value of .026, computational errors have a statistically significant influence on students' performance in logarithms in public secondary schools in Mwala Sub-county.

The study concludes that the statistical significance of mechanical errors on students' performance in logarithms is confirmed by a p-value of .015 from the ANOVA results, indicating these errors considerably affect students' understanding of the topic.

RECOMMENDATIONS

The study recommends employing error analysis as a valuable tool in education, allowing teachers to use mistakes as opportunities for targeted intervention to decrease their occurrence. Specifically, teachers could enhance learning by presenting both incorrect and correct examples of logarithmic calculations in the classroom, thus exposing students to common mistakes and how to avoid them. Additionally, it is advised that teachers pay close attention to their communication methods, including the timing, wording, and presentation of logarithmic concepts on the board, to prevent misunderstandings that could lead to errors.

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